



Kim Williams
Michael J. Ostwald
Editors

Architecture and Mathematics from Antiquity to the Future

Volume I:
Antiquity to the 1500s

 Birkhäuser

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Kim Williams
Kim Williams Books
Torino, Italy

Michael J. Ostwald
School of Architecture and Built Environment
The University of Newcastle
Callaghan, Australia

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Giza, Egypt. Photograph © Giulio Magli, reproduced by permission.

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Preface to Architecture and Mathematics from Antiquity to the Future

In June of 1996, in his keynote address at the conference ‘Nexus’96: Relationships Between Architecture and Mathematics’, the founding international conference of what would become an international community for research in a new interdisciplinary field, eminent engineer Mario Salvadori asked, ‘[c]an there be any relationship between architecture and mathematics?’ Over the next 18 years, the Nexus community came together for a series of bi-yearly conferences in Italy, Portugal, Mexico, Turkey and the USA to examine, debate and celebrate the relationships that exist between architecture and mathematics. The conferences were hosted in locations where important historic connections had been proposed between architecture and mathematics: in Europe these locations include Fucecchio (1996), Mantua (1998), Ferrara (2000), Óbidos (2002), Genoa (2006), Porto (2010) and Milan (2012). Further afield, conferences were held in Mexico City (2004), San Diego (2008) and Ankara (2014). Conference venues were chosen to permit participants to visit local sites of historic importance for architecture and mathematics in post-conference workshops, such as Pompeii and Herculaneum in 1996, the villas of Palladio in 1998 and Teotihuacan in 2004. The speakers at these events include some of the most influential people in architecture, art, mathematics and engineering. Lionel March, Robert Tavenor, Alberto Pérez-Gómez, Marco Frascari, Michele Emmer, Leonard Eaton and Mario Salvadori, amongst many other luminaries, have all presented at the Nexus conferences and taken part in round-table discussions, forums and visits to some of the great architecture of these regions.

The first Nexus conference was actually conceived out of the frustration caused by the difficulty of finding a venue for publishing interdisciplinary research: papers in architecture and mathematics were seen as too mathematical for architectural journals, but not mathematical enough for mathematics journals. At best, such research was viewed as a curiosity, too far from the mainstream to garner much interest. Because there was no single journal that encouraged such research, when authors were fortunate enough to have an article accepted, publications were scattered, and authors seldom knew about the work of others examining similar topics. The Internet was in its infancy at that time, leaving far-flung scholars to

work in isolation. One journal, *The Mathematical Intelligencer*, and its particularly open-minded editor-in-chief, Chandler Davis, had accepted papers by three of the participants at the first conference, Kim Williams, Benno Artmann and Heinz Götze, who subsequently began to correspond. The 23 people who met in 1996 at the first conference knew of each other's work by word of mouth: friends sending their work to friends. But already by the second conference, 2 years later, the growing group felt the need for a publishing venue, and it was decided to found the *Nexus Network Journal*—*Nexus*, from the name chosen for the first conference to represent the idea of interweaving ideas from two disciplines, and *Network*, to describe the group of people whose acquaintances and collaborations were continuing to expand. The first issue of the journal, with Kim Williams as editor-in-chief, was introduced online in 1999, was added to at trimester intervals of the course of that year and was produced in print at its end. The journal continued in that way for its first 2 years, but by volume 3 in 2001, submissions had grown so much that it was published in two issues per year, until with volume 9 in 2010, it grew to three issues per year.

Across 15 volumes, 35 issues and over 500 refereed papers, the international reputation and impact of the journal have grown considerably. Now published jointly in the Birkhäuser programme of Springer-Basel and Kim Williams Books of Torino, Italy, the journal is highly respected and has a growing readership. Beginning with volume 16 in 2014, the *NNJ* will be overseen jointly by the editors of these present two volumes.

Foreseen along with the conferences was the publication of the proceedings. The series entitled 'Nexus: Architecture and Mathematics' comprised seven volumes from the first seven conferences. At the beginning, the conference books were seen as separate from the journal. This changed with the eighth conference, when speakers voiced the desire to see their papers published in the *NNJ*, which was by that time mature and esteemed. Thus, since 2010, papers presented in the Nexus conferences have been published in special issues of the journal and are available online. However, the research presented at the early conferences was only available in a series of limited edition books. With many of these being out of print there has been growing pressure to make the most highly cited works from the early years of the Nexus conferences available. Rather than simply republishing selected works in the order in which they were written, such was the scope of these early Nexus publications that an alternative proposition presented itself.

We, the editors, have assembled almost a hundred papers from the early years of the Nexus conferences, and arranged them both thematically and chronologically to trace key moments in the history and theory of architecture and mathematics, from antiquity to the present day, along with predictions for the future. These chapters describe over 60 major buildings and architectural works, consider more than twenty major theories of geometry and design and cover themes and ideas arising from five continents and spanning over four millenia.

Having said this, the present two-volume work does not pretend to be a comprehensive encyclopaedia of the history and theory of every facet of the relationship between architecture and mathematics. Being works by more than

one hundred authors with backgrounds in not only architecture and mathematics but also engineering, physics, chemistry, philosophy, music and more, there is a rich diversity of approaches to the topic, along with some insightful synergies and informative disagreements. All of the chapters have undergone minor editorial revisions, including, in some cases, updated bibliographies. In a few cases authors have chosen to make more substantial revisions, to bring their chapters up to date, or direct the reader to advances that are currently occurring in their areas. In addition to this, we have provided an overview chapter for each volume (Chap. 1 in vol. I and Chap. 48 in vol. II), to frame the sequence and structure of the whole as well as a chapter entitled 'Mathematics *in, of and for* Architecture: A Framework of Types' (Chap. 3) which seeks to classify, and thereby make more accessible, the myriad connections proposed across this work.

Each of the chapters in the present work have become crucial landmarks in the scholarly landscape of architecture and mathematics. Some represent pioneering research, the first studies of the relationships between architecture and mathematics in a specific period, or in the oeuvre of a given architect. They serve as both points of departure for new voyages of discovery and as destinations for people entering unfamiliar terrain. For the novice researcher these works provide a grounding for their explorations, and for seasoned scholars these chapters offer a critical record of the efforts of fellow travellers. We, the editors, hope that through this two-volume work, these chapters can continue to inspire and guide future generations.

We wish to thank Maria Roberts, Valentina Filemio and Marco Giorgio Bevilacqua for assistance with editing and proofing, and Michael Dawes for support with image preparation and research assistance. We thank all authors for permission to reuse their material, and for their help in updating texts and references. Finally, we thank Anna Mätzener, Editor for Mathematics and History of Science, and Thomas Hempfling, Executive Editor for Mathematics, Birkhäuser, for their support of the Nexus conferences and the *Nexus Network Journal* throughout the years, and especially for their support of this present work.

Torino, Italy
Newcastle, Australia
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Kim Williams
Michael J. Ostwald

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Contributors to Volume I

Benno Artmann (1933–2010)

Niels Bandholm Kloster, Ringkoebing, Denmark

Javier Barrallo School of Architecture, The University of the Basque Country (UPV-EHU), San Sebastian, Spain

Maria Teresa Bartoli Dipartimento di Architettura, Università degli studi di Firenze, Florence, Italy

Avril Behan Dublin Institute of Technology, College of Engineering & Built Environment, Dublin, Ireland

Gerardo Burkle-Elizondo Centro Interinstitucional de Investigaciones en Arte y Humanidades, Universidad Autónoma de Zacatecas, Zacatecas Zac, Mexico

Paul A. Calter Vermont Technical College, Randolph Center, VT, USA

Luisa Rossi Costa Department of Mathematics, Politecnico di Milano, Milan, Italy

Salvatore di Pasquale (1931–2004)

Yvonne Dold-Samplonius Ruprecht-Karls-Universität Heidelberg, Institute of Mathematics, Heidelberg, Germany

Sylvie Duvernoy Politecnico di Milano, Milan, Italy

Holger Falter Hochschule für angewandte Wissenschaften Coburg, Coburg, Germany

Marco Frascari (1945–2013)

Heinz Götze (1912–2001)

Paulus Gerdes (1952–2014)

Gulzar Haider School of Architecture, Beaconhouse National University, Lahore, Pakistan

István Hargittai Materials Structure and Modeling Research Group, Budapest University of Technology and Economics, Budapest, Hungary

Magdolna Hargittai Materials Structure and Modeling Research Group, Budapest University of Technology and Economics, Budapest, Hungary

Silvia L. Harmsen St. Leon-Rot, Germany

George Gheverghese Joseph University of Manchester, Manchester, UK

Jay Kappraff Department of Mathematics, New Jersey Institute of Technology, University Heights, Newark, NY, USA

Izumi Kuroishi School of Cultural and Creative Studies, Aoyama Gakuin University, Shibuya-ku, Tokyo, Japan

Andrew I-kang Li Tokyo, Japan

Elena Marchetti Department of Mathematics, Politecnico di Milano, Milan, Italy

Rafael Ramírez Melendez Pompeu Fabra University, Barcelona, Spain

Rachel Moss Department of the History of Art and Architecture, Trinity College, Dublin 2, Ireland

Muhammad Moussa HHCP Architects, Orlando, FL, USA

Vini Nathan College of Architecture, Design and Construction, Auburn University, Auburn, AL, USA

Michael J. Ostwald School of Architecture and Built Environment, University of Newcastle, Callaghan, NSW, Australia

Alpay Özdural (1944–2003)

Alfonso Ramírez Ponce E 21, M XII Educación, Coyoacán, México, D.F.

Graham Pont Balmain, NSW, Australia

Mark A. Reynolds Academy of Art University, San Francisco

Clara Silvia Roero Department of Mathematics “Giuseppe Peano”, Università di Torino, Turin(Torino), Italy

Nicoletta Sala Accademia di Architettura, Università della Svizzera italiana, Mendrisio, Switzerland

Peter Saltzman Berkeley, CA, USA

Mario Salvadori (1907–1997)

Santiago Sanchez-Beitia School of Architecture, The University of the Basque Country (UPV-EHU), San Sebastian, Spain

William D. Sapp San Bernardino National Forest, San Bernardino, CA, USA

Peter Schneider College of Architecture and Planning, University of Colorado, Denver, USA

David Speiser Université catholique de Louvain, Louvain-La-Neuve, Belgium

Gert Sperling Vellmar, Germany

Rudolf H.W. Stichel Fachbereich Architektur, Technische Universität Darmstadt, Darmstadt, Germany

Helge Svenshon Fachbereich Architektur, Technische Universität Darmstadt, Darmstadt, Germany

Robert Tavernor Tavernor Consultancy, Pimlico, London, UK

Ricardo David Valdez-Cepeda Centro Regional Universitario Centro Norte Apdo, Universidad Autónoma Chapingo, Zacatecas Zac, Mexico

Livio G. Volpi Ghirardini Accademia Nazionale Virgiliana, Mantova, Italy

Stephen R. Wassell Department of Mathematics and Computer Science, Sweet Briar College, Sweet Briar, VA, USA

Carol Martin Watts The College of Architecture, Planning & Design, Kansas State University, Manhattan, KS, USA

Kim Williams Kim Williams Books, Turin (Torino), Italy

Mark Wilson Jones Department of Architecture and Civil Engineering, University of Bath, Bath, UK

Michael R. Ytterberg BLT Architects, Philadelphia, PA, USA

Radoslav Zuk School of Architecture, McGill University, Montreal, QC, Canada

Chapter 1

Relationships Between Architecture and Mathematics

Michael J. Ostwald and Kim Williams

architecture *l'ærki, tek ch ørl noun*

The art or practice of designing and constructing buildings. The style in which a building is designed or constructed, esp. with regard to a specific period, place, or culture.

mathematics *lmaθ(ə)'matiks/ plural noun*

The abstract science of number, quantity, and space. Mathematics may be studied in its own right (pure mathematics), or as it is applied to other disciplines such as physics and engineering (applied mathematics).

Introduction

What is the nature of the relationship between architects and mathematicians or between architecture and mathematics? As they are commonly understood these two groups seem to have few obvious connections. The word ‘architecture’ is used to describe either the practice of creating buildings or a particular class of constructed—architectural—objects. In contrast, the word ‘mathematics’ denotes a domain of pure or applied knowledge that is associated with the study or use of abstract objects such as numbers and shapes or forms. Professions, like architecture, tend to be isolated and controlled, restricting membership to experts who have been awarded particular qualifications and have fulfilled certain criteria (Fournier 2000).

M.J. Ostwald

School of Architecture and Built Environment, University of Newcastle, Callaghan,
New South Wales 2308, Australia

e-mail: michael.ostwald@newcastle.edu.au

K. Williams (✉)

Kim Williams Books, Corso Regina Margherita, 72, 10153 Turin (Torino), Italy

e-mail: kwb@kimwilliamsbooks.com

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Instead, disciplines, like mathematics, are ways of grouping and identifying bodies of knowledge and expertise that are both pertinent within the discipline and which might also be applied in other fields (Klein 1996). However, despite these apparent differences, the distance between these two—between a profession and a discipline and between an object (of design) and a subject (of study)—is far less than many would assume. In order to make sense of the contemporary view which considers architecture and mathematics as dissimilar pursuits, it is useful to trace a brief history of the growth of disciplines and professions over time; a history which, most importantly, reminds us that in the past these two were very closely connected.

In the world of ancient Greece, learned professionals were trained in one of three fields: religion, ethics, or the human condition. While some distinction was made between the ways students were educated in each of these fields, a sound foundation in both philosophy and mathematics was considered a necessity (Marrou 1956). At the same time, the acts of creating a chair, designing a house or constructing a ballista were all tasks that could be undertaken by artisans. Such skilled workers were trained in the practical processes of making (stone-cutting, woodwork) along with a range of knowledge domains including both geometry and metrology (Kostof 1977a).

Later, in Roman times, Vitruvius tells us that the architect was neither a scientist nor a craftsman, but one who had sufficient knowledge of a range of scientific fields—first and foremost geometry, but also history, philosophy, music, medicine, law and astronomy—to be able to oversee the work of all other disciplines (Vitruvius 2009: 5).

A similar pattern of relationships is found in medieval Europe where the first universities recognised graduates in law, divinity and medicine. In the medieval university students were expected to complete preparatory studies on the *trivium*—grammar, logic and rhetoric—along with studies in arithmetic, geometry and aesthetics (Janin 2008). Seated in rows of carved pews, in high-roofed tapestry-lined halls, these students were able to directly observe the geometric mysteries of space and form. But the very halls they inhabited during these studies had been created by master builders and teams of craftsmen and labourers with a fund of geometric knowledge. Evidence of this knowledge is found in Villard de Honnecourt's *Livre de Portraiture*, where he wrote, “. . . in this book . . . you are able to find the technique of representation as the discipline of geometry requires and instructs” (Barnes 2009: 35). The most senior of the master builders, often the direct descendants of artisan families, were given the title ‘architect’ or ‘engineer’ (Kostof 1977b). These master builders were trained in both the arts and sciences of design and construction. Furthermore, in both the Classical and Medieval eras, the mathematical disciplines provided a critical foundation for construction (Fitchen 1961; James 1981). This is why, for much of this era, there was little distinction between professions and disciplines, or between architects and mathematicians. This close and productive relationship between architecture and mathematics was to continue for several centuries before reaching its most visible apogee during the Renaissance.

During the Renaissance the Medieval educational foundation found in the *trivium* was expanded to encompass a second tier of four arts (the *quadrivium*) that typically included studies of arithmetic, geometry, music and astronomy. The archetypical ‘Renaissance man’ was expected to have both a broad and comprehensive knowledge across these seven subjects, along with their potential application in at least one, and possibly more, of the following pursuits; science, art, medicine and architecture. Leon Battista Alberti was one such polymath; an author, artist, poet and linguist who had also mastered optics, perspective and cryptography (Williams et al. 2010). Today, Alberti is best known as an architect, because it was in the application of these ideas to design, in both written and built work, that he achieved his most enduring success. For a similar reason Christopher Wren is also regarded as one of the world’s great architects, even though he originally distinguished himself in astronomy, physics and mathematics and only began to design buildings when he was already a respected scientist (Bennett 1982). Robert Hooke is another example of the natural philosopher-cum-architect. Even the great Isaac Newton demonstrated an amateur, but informed, interest in architectural theory. Such cases suggest that for centuries architecture and mathematics were closely related, and equally respected, areas of inquiry.

However, despite the examples of Wren and others, the seventeenth century was more generally marked by the rise in power of guilds and colleges who sought to define and preserve their members’ interests (Melton 2001). For example, it was around this time that the Freemasons formulated a series of rules of membership and practice that sought to protect the knowledge and skills of the stonemasons. Amongst the earliest articles of Freemasonry is a set of practical and symbolic rules showing the essential relationship between architecture and geometry (Berman 2012). Thus, while the tradition of the ‘Renaissance man’ was still being valorised in the eighteenth century, the guilds and technical colleges remained the driving force which gradually separated professions from disciplines and, inadvertently, increased specialisation began to distance architecture from mathematics (Clarke 1994).

Over the ensuing 200 years, in parallel with the emergence of new technology and the need for more focussed trades and skills, disciplines and professions became increasingly specialised and their roles began to change (Duffy and Hutton 1998). For example, established in 1794 the French *École Polytechnique* was at the forefront of approaches to training a new, elite class of technocrats. Embracing a scientific disposition, the curriculum included issues of aesthetic perception, positivism and rationalism. In part because of this educational focus, this era marked a growing separation between architects and engineers (Picon 1992). The French Enlightenment was also one of the last periods wherein architects, still trained in descriptive geometry, directly contributed to mathematical knowledge. In particular, the discipline of stereotomy was developed largely by architects to allow stone blocks to be cut and assembled in complex forms (Warren 1875).

By the early years of the twentieth century architectural education was split between an atelier-based model, which traced its origins to the *École des*

Beaux-Arts, and the technical college model, which acknowledged a lineage to the apprenticeship system (Cuff 1991; Crinson and Lubbock 1994). The atelier system, modelled on fine art practice, included geometry in its core curriculum but the role of arithmetic was less apparent (Draper 1977). The technical college system included both science and mathematics in its syllabus along with more extensive applications of geometry. Significantly though, despite the apparent differences between these systems, both effectively positioned mathematics as a secondary discipline which merely served to buttress the education of architects (Boyer and Mitgang 1996). In a comparable way, the discipline of mathematics divided itself into a 'pure' and an 'applied' strand, with the former being regarded as the path for specialists, and the latter for those who sought to engage more directly with other fields (Davis et al. 1995). This pattern was repeated around the world with the combination of increasing specialisation and the desire for professional recognition gradually separating and isolating different knowledge domains and disciplines from each other (Fournier 2000).

Such was the compartmentalisation of knowledge that occurred in the early part of the twentieth century that British scientist and novelist Charles Percy Snow famously criticised the rise of two distinct and separate cultures—science and humanities—each seemingly unaware of the basic values and lessons of the other (Snow 1998). Snow's observations, derived from his identification of the growing separation between disciplinary groups, were both widely reported and criticised (Carafoli et al. 2009). Certainly it was becoming harder for a person to be qualified in two or more fields and the era of the peripatetic scholar was effectively over. This was true even within the discipline of mathematics itself; Henri Poincaré was considered the "last universal mathematician". Yet, the growing accessibility of knowledge, no longer protected by professional guilds or enshrined in the lore of esoteric societies, meant that rather than fostering the divide between two distinct cultures, there were potentially a multitude of secondary connections to be made between different groups, each creating new sub-cultures (Nicolescu 2002). However, such new transdisciplinary groups face a twofold problem: visibility and recognition (Doucet and Janssens 2011). In the first instance, while important connections exist between fields, like architecture and mathematics, they are often rendered invisible by contemporary educational practices and the legal implications of professional ethics (Sokolowski 1991). In the second, sustained research must take place before such hidden associations can be recognised, investigated and celebrated. This is especially the case in contemporary society where these same two problems of visibility and recognition continue to hinder our capacity to engage with transdisciplinary knowledge.

Today, the extent to which architects are formally trained in mathematics is probably lower than in any previous period in history (Ostwald and Williams 2008). The degree to which mathematicians directly engage with building design and construction is at an equally low ebb. This is unfortunately true of the general public as well, since no formal architectural education at all is offered in public schools, while at least of minimum of mathematics is taught. Yet, this situation is not entirely as it seems. Advances in computing have placed mathematical techniques and processes at the fingertips of every young architect in the world,

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